Technical Notes

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Turbulent Boundary-Layer Shear Stress Distributions for Compressible Adverse Pressure Gradient Flow

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Nomenclature

p = pressure
u = streamwise velocity component
v = velocity component normal to the local surface
x = streamwise coordinate
y = coordinate normal to the local surface
δ = boundary-layer thickness
δ_u = boundary-layer velocity thickness
θ = boundary-layer momentum thickness
κ = inverse longitudinal curvature, $1/R$
μ = molecular viscosity
ρ = density
τ = shear stress

Subscripts

w = property evaluated at the wall
∞ = reference condition, property evaluated at $y = \delta$

SINCE no direct measurements of the turbulent shear stress distribution in the supersonic turbulent boundary layer had been reported until the recent measurements of Rose,[1] models of the turbulent shear stress distribution for compressible flow have been obtained by extending results from the more thoroughly measured incompressible turbulent boundary layer. Calculation procedures employing models of the turbulent shear stress established from incompressible experiments have yielded good agreement with experimental data for zero pressure gradient or mildly favorable pressure gradient, adiabatic flow. However, less favorable results have been obtained when computing flow in which adverse pressure gradient and longitudinal curvature are encountered.

This Note describes calculations of turbulent shear stress, mixing length, and eddy viscosity distributions using supersonic nozzle wall boundary-layer mean profile data for isentropic ramp induced adverse pressure gradient flow.[2] Similar calculations applied to experimental data for this flow configuration have not been previously reported.

These calculations have been made as part of an effort to develop an improved model of the eddy viscosity for use in numerical computation of turbulent boundary-layer development. These calculations are also to be used for comparison with direct measurements of the turbulent shear stress when the measurements are completed.

The experimental data used in these calculations[2] were obtained in the nozzle wall boundary layer of Supersonic Wind Tunnel No. 2 of the Ballistic Research Labs. This is a continuous operating facility with an asymmetric flexible plate nozzle. Figure 1 shows a schematic of the nozzle and the configuration of the test section. The model used to create the region of adverse pressure gradient had a gradually changing radius of curvature that varied from 81.3 cm at the front to 100.6 cm at the rear. The boundary layer, which had developed naturally along the smooth flat surface of the nozzle wall, had the following characteristics in the zero pressure gradient flow at station 10 immediately upstream of the ramp model

$$Re_\theta = 2.825 \times 10^4, \quad M_\infty = 3.51, \quad T_w/T_{t_\infty} = 0.94$$
$$\delta = 2.40 \text{ cm}, \quad \delta^* = 0.872 \text{ cm}, \quad \theta = 0.133 \text{ cm}$$

The equations for conservation of mass and streamwise momentum as applicable to two-dimensional, compressible turbulent boundary-layer flow over a surface with longitudinal curvature may be integrated in the y direction normal to the

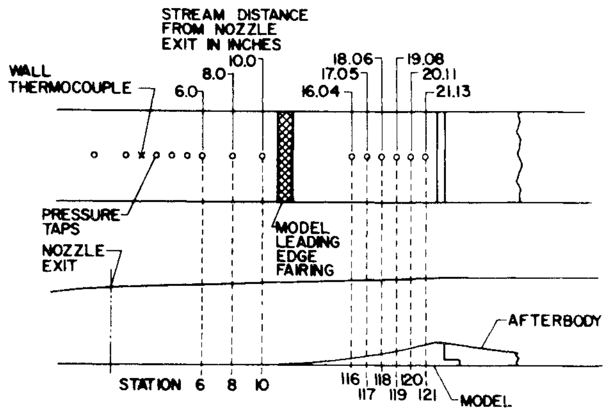

Fig. 1 Schematic of tunnel nozzle test section configuration and test station identification.

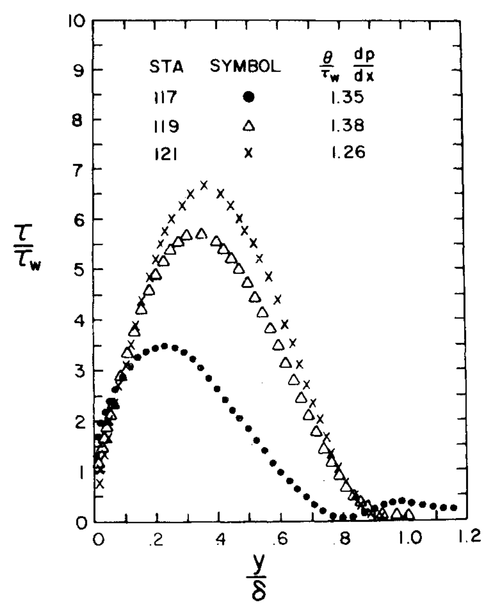

Fig. 2 Calculated shear stress profiles for flow over the ramp model.

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local surface and combined to yield the following relation for the shear stress distribution[3]

$$\frac{\tau}{\tau_w} = \frac{1}{\tau_w}\left\{\tau_w + \int_0^y \beta\frac{\partial}{\partial x}(\rho u^2)\,dy - u\beta\int_0^y \frac{\partial}{\partial x}(\rho u)\,dy - 2\int_0^y\left[\int_0^y \frac{\partial}{\partial x}(\rho u)\,dy\right]u\beta^2\kappa\,dy + \int_0^y \beta\frac{\partial p}{\partial x}\,dy\right\} \quad (1)$$

where $\beta = 1/(1+\kappa y)$ and $\tau = \mu(\partial u/\partial y) - \rho\langle u'v'\rangle$. This relation has been integrated directly using measured values of wall shear stress and tabulated profile data. Values of the partial derivatives have been determined along lines of constant mass flux using a least squares technique.

The shear stress profiles obtained are shown in Fig. 2. These profiles are consistent with the boundary condition at the wall, $d\tau/dy = dp/dx$. The trend of the shear stress profile to increase beyond the point of minimum shear stress is inconsistent with the requirement that τ approach zero in the vicinity of the edge of the boundary layer. This trend is believed to be a result of the uncertainty in determining the streamwise partial derivatives.

The trend of the maximum value of τ to increase at downstream stations has been observed in incompressible flow; however, not to the extent observed here.

The eddy viscosity and mixing length distributions were calculated using the relations

$$\frac{l}{\delta} = \frac{1}{\delta}\left(\frac{\tau}{\rho(du/dy)^2}\right)^{1/2} \quad (2)$$

$$\frac{\varepsilon}{u_\infty \delta_u} = \frac{1}{u_\infty \delta_u}\left(\frac{\tau}{\rho(du/dy)}\right) \quad (3)$$

The local value of density was obtained from the tabulated data, and the velocity derivative, du/dy, was determined by central differentiation of the velocity profile data. Examples of the mixing length distributions obtained are shown in Fig. 3. These plots exhibit a linear region near the wall with a slope of 0.65 rather than the well accepted value for zero pressure gradient flow of 0.4. Also, the peak value is seen to increase as the flow develops over the ramp model in agreement with the trend of the shear stress distribution.

The eddy viscosity distributions obtained are shown in Fig. 4. The eddy viscosity is seen to exhibit a behavior similar to that for zero pressure gradient flow with the exceptions: 1) the magnitude of the peak value is greater than that obtained for zero pressure gradient flow, and 2) the peak value increases as the flow develops over the ramp model.

The calculated distributions of mixing length and eddy viscosity for adverse pressure gradient flows have been shown to be considerably distorted when compared to their zero pressure gradient counterparts. Although velocity profiles calculated for adiabatic, nonporous walls by numerical programs exhibit only a moderate sensitivity to the eddy viscosity model employed, it is conceivable that use of the eddy viscosity models calculated here could have substantial impact.

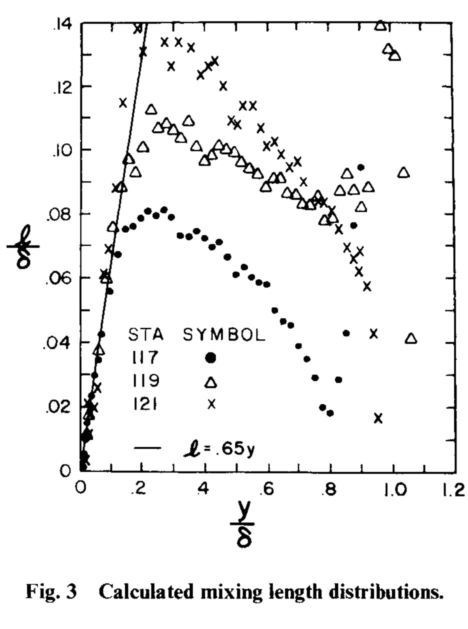

Fig. 3 Calculated mixing length distributions.

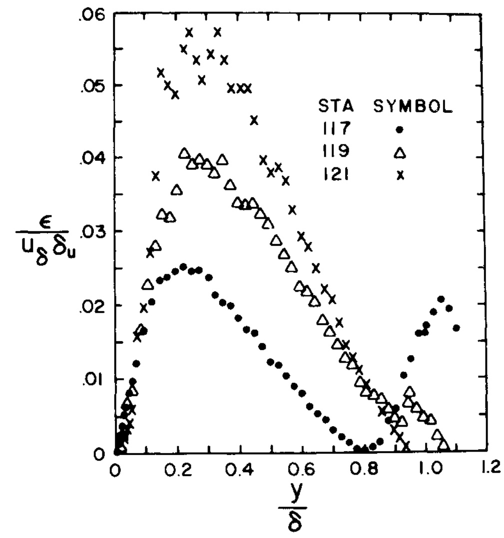

Fig. 4 Calculated eddy viscosity distributions.

References

[1] Rose, W. C., "The Behavior of a Compressible Turbulent Boundary-Layer in a Shock-Wave-Induced Adverse Pressure Gradient," Ph.D. dissertation, 1972, Univ. of Washington, Seattle, Wash.

[2] Sturek, W. B. and Danberg, J. E., "Supersonic Turbulent Boundary Layer in Adverse Pressure Gradient. Part I: The Experiment," *AIAA Journal*, Vol. 10, No. 4, April 1972, pp. 475–480.

[3] Sturek, W. B., "Calculations of Turbulent Shear Stress in Supersonic Turbulent Boundary-Layer Zero and Adverse Pressure Gradient Flow," BRL Rept. 1651, AD 763197, June 1973, U.S. Army Ballistic Research Labs., Aberdeen Proving Ground, Md.

Calculations of Flow Around a Semi-Infinite Flat Plate in a Shock Tube

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Introduction

RECENT progress in digital computers has made the solutions of problems involving viscous compressible flows possible by means of difference methods. In this Note, we solve the case of a shock-induced unsteady laminar boundary layer on a semi-infinite flat plate without adopting the assumption[1,2] that the boundary-layer approximations are valid. In order to integrate the two-dimensional, unsteady, compressible Navier-Stokes (N-S) equations, we adopt two difference schemes; the Thommen method[3] or scheme 1 and a modified form of the Saul'ev method,[4] scheme 2, obtained by applying the Saul'ev method to the parabolic part of the N-S equations. It is found that the flowfield patterns obtained by these methods are similar,

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